VOLUME 76 SEPARATE No. 4

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

FEBRUARY, 1950

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

CAPILLARY PHENOMENA IN COHESIONLESS SOILS

By T. William Lambe, Jun. ASCE

SOIL MECHANICS DIVISION

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

PRICE $0.50 PER COPY

Published at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

CAPILLARY PHENOMENA IN COHESIONLESS SOILS

By T. William Lambe,[1] Jun. ASCE

Synopsis

This paper explains the fundamentals of capillarity in cohesionless soils, by first describing analogies in capillary tubes and then presenting data from soil tests to substantiate the conclusions drawn from the analogies. A number of theoretical methods, in common use, for computing the rate of flow of water under the influence of capillarity are investigated and evaluated. Improvements in several of these theories are suggested.

The limiting values of capillary head which a cohesionless soil may have are presented and discussed. The role of the different capillary heads in the various types of capillary flow are also explained. Names are suggested for these capillary heads.

Introduction

In the design of roads and airport runways, one of the most important considerations is the proper control of surface and subsurface water. Likewise, the design of earth dams, earth slopes, and walls that retain soil backfills may be controlled by the water that passes through and remains within the soil in question. In addition to its influence on man's structures, water governs the growth of nature's plants since they rely on the ground water their roots supply to them. The movement and retention of this important soil water can be largely dependent on the phenomenon of surface tension as exhibited in capillaries.

Capillarity, as used in soil mechanics, is that property which enables the soil to draw and hold water above the elevation at which atmospheric pressure exists in the water. The importance of capillarity in soils has been realized for many years by agricultural scientists as well as by engineers who design highways, airports, dams, retaining walls, and foundations. As a result, a large amount of research on soil capillarity has been reported, but the purpose of

Note.—Written comments are invited for publication; the last discussion should be submitted by **July 1, 1950.**

[1] Asst. Prof. of Soil Mechanics, Massachusetts Inst. of Technology, Cambridge, Mass.

nearly all this study has been to obtain numerical measures of capillarity on many different soils.

Some incorrect concepts appear in the literature because of the incomplete fundamental understanding of the role of capillarity in the movement and retention of soil water. Therefore, it is the purpose of this paper to help explain the fundamentals of capillarity in cohesionless soils and to evaluate the assumptions involved in some of the current theories of capillary water in soils. In addition, improvements in several of the theories will be suggested.

All experimental work reported in this paper was done on one soil under test conditions that were made as similar as possible. The soil used was a fine, uniform, natural sand which had a 60% size of 0.19 mm and a 10% size of 0.08 mm. Since only one soil and one set of test conditions were used, the numerical results of the laboratory tests have meaning only for this case.

Capillary Flow into Cohesionless Soil

Description of Horizontal Capillary Flow.—To explain the horizontal movement of water in soils under the pull of capillarity, a study of flow in capillary tubes will first be made. Fig 1 shows a horizontal capillary system comprised of tubes 1 and 2, which have no interconnection. The tubes have diameters, D, and capillary heads, h, as shown. (All pressures considered in this paper are based on atmospheric pressure taken as zero pressure.)

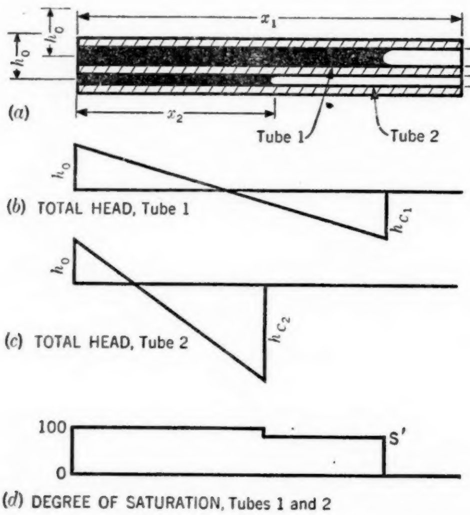

Fig. 1.—Flow in Independent Horizontal Capillary Tubes

By Poiseuille's law, the rate of flow, q, in a tube is given[2] by

$$q = \frac{\gamma r^2}{8 \mu} i A \ldots\ldots (1)$$

in which γ is the unit weight of water; r is the radius of the tube; μ is the viscosity of the flowing water; i is the gradient; and A is the cross section of the area.

Since for each tube the expression $\frac{\gamma r^2}{8 \mu}$ is constant, it may be taken equal to k', which corresponds to the Darcy coefficient of permeability, k. Eq. 1 can be written

$$q = k' i A \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

[2] "Fundamentals of Soil Mechanics," by D. W. Taylor, John Wiley & Sons, Inc., New York, N. Y., 1948.

When the meniscus has moved a distance of x from the left end,

$$i = \frac{h_o + h_c}{x} \qquad (3)$$

Substitution of Eq. 3 in Eq. 2 gives

$$q = k' \frac{h_o + h_c}{x} A \qquad (4a)$$

Since the flow is continuous, the rate must also be equal to velocity of movement of the wetted surface, $\gamma = \frac{dx}{dt}$, multiplied by the area, or

$$q = \frac{dx}{dt} A \qquad (4b)$$

Equating Eqs. 4a and 4b gives

$$q = k' \frac{h_o + h_c}{x} A = \frac{dx}{dt} A \qquad (4c)$$

Solution of Eq. 4c gives

$$x = \sqrt{2\,k'\,t\,(h_o + h_c)} \qquad (5)$$

By using Eq. 5, one can locate the wetted surface at any time in either tube of the system in Fig. 1. For example, let $D_1 = 2\,D_2$; and $h_o = h_{c1}$. For this specific example, since h_c varies[3] as $\frac{1}{D}$, solution of Eq. 5 gives

$$x_1 = \sqrt{\frac{8}{3}}\,x_2 = 1.63\,x_2 \qquad (6)$$

Thus, in this example the air-water interface in tube 1 is always 1.63 times as far from the left end as that in tube 2.

Although the foregoing analogy illustrates some useful concepts applicable to soils, there is a most important difference between system 1 in Fig. 1 and the capillary systems of soils—in soil there exists an infinite number of interconnections between the various effective capillary tubes.

To simulate the action in soils more closely, the system of Fig. 1 will be modified by interconnecting the two tubes at many points along their length. No difference in total head can now exist between the tubes because the interconnections would permit cross flow to equalize any tendencies toward such a difference. Therefore, at any distance less than the x of the lagging meniscus, the total head is the same in the two tubes. Fig. 2 shows the modified system in which the movement of water has progressed to x_1 in the large tube and to x_2 in the small tube. Fig. 2(b) illustrates that the heads in the two tubes are alike at any distance for 0 to x_1; therefore, it follows that the gradient in this range is the same in both tubes.

An important difference between the systems of Figs. 1 and 2 is that in the latter the meniscus in the small tube precedes that in the large one by a distance

[3] "Colloid and Capillary Chemistry," by Herbert Freundlich, Methuen and Co., Ltd., London, 1926.

$x_2 - x_1$. The meniscus in the large tube cannot precede that in the small tube because flow would occur from the large tube to the small tube, as a result of the higher negative pressure in the small tube, until the meniscus of the large tube was lagging. In addition, the value of x_2 can never be such that the gradient over this distance is constant because this would mean that, although the tubes, in effect, had no influence on each other, the value of x_2 would be greater than x_1. The system in Fig. 1 shows this latter condition to be untrue. Therefore, x_2 must be greater than x_1 but can never be large enough for a constant gradient throughout its length. (The foregoing qualitative reasoning is sufficient for the purposes of this analogy. A more exact study of the problem shows that at x_1 there is a small difference of head between the small and the large tubes, as at point M, Fig. 2(b). There is flow from the large into the small tube at point M due to this head difference. This cross flow decreases the gradient in the large tube at x_1, thus reducing the velocity of the meniscus in the large tube. A solution to the problem indicates that the distance between the menisci, $x_2 - x_1$, increases with an increase of x_1 and decreases with an increase in cross flow.)

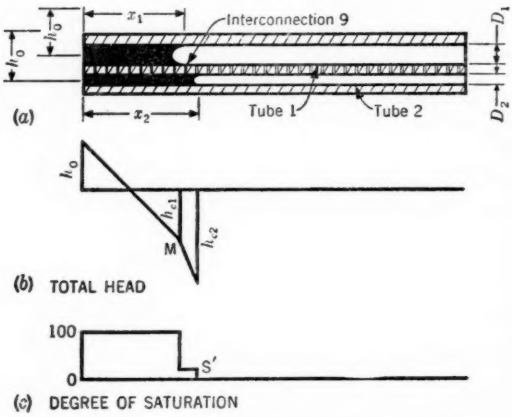

Fig. 2.—Flow in Interconnected Horizontal Capillary Tubes

The flow in the system of Fig. 2 has a number of characteristics which have counterparts in capillary flow through soils, the most important of which are:

1. The meniscus in the smaller tube precedes that in the larger tube;
2. The only tube diameter that is effective at any instant and at any point in developing capillary head is that diameter where a meniscus is formed;
3. The gradient in both tubes back of the air-water interface in the large tube is essentially constant, because lateral flow between the tubes equalizes heads;
4. The average gradient for the system depends more on the larger tube, since a large part of the greater capillary head of the smaller tube is lost in the distance between the two menisci; and
5. In this zone between the two menisci, k' (which corresponds to permeability in soils) and the degree of saturation for the system are lower than those back of the lagging meniscus.

Some of the more important inherent differences between soils and a system of capillary tubes, which make their flow processes unlike, are:

(a) In the system in Fig. 2, the sole function of the interconnecting tubes was to permit sufficient flow to equalize pressure in the two tubes. Since one

water surface always preceded the other, the connections could always fill themselves by capillarity (see interconnection 9, Fig. 2(*a*)). This ideal situation does not exist in soils, since there may be two pores that have the same effective diameter and thus become filled simultaneously; therefore, a small connecting tube between these pores would not be saturated. In such a connecting tube, flow would proceed from one or both ends until the air pressure balanced the capillary head and stopped the flow. As a result of this and other air entrapping phenomena in soils, the maximum degree of saturation attained by capillary action is less than 100%.

(*b*) In soils, the equivalent capillary tubes change diameters from point to point. (Microscopic study by the writer showed capillary flow to be very jerky. This sudden change of velocity is due to sudden changes in void sizes along the path of flow.)

(*c*) There is an almost infinite number of effective tube sizes in soils rather than the two considered in the previous analogies.

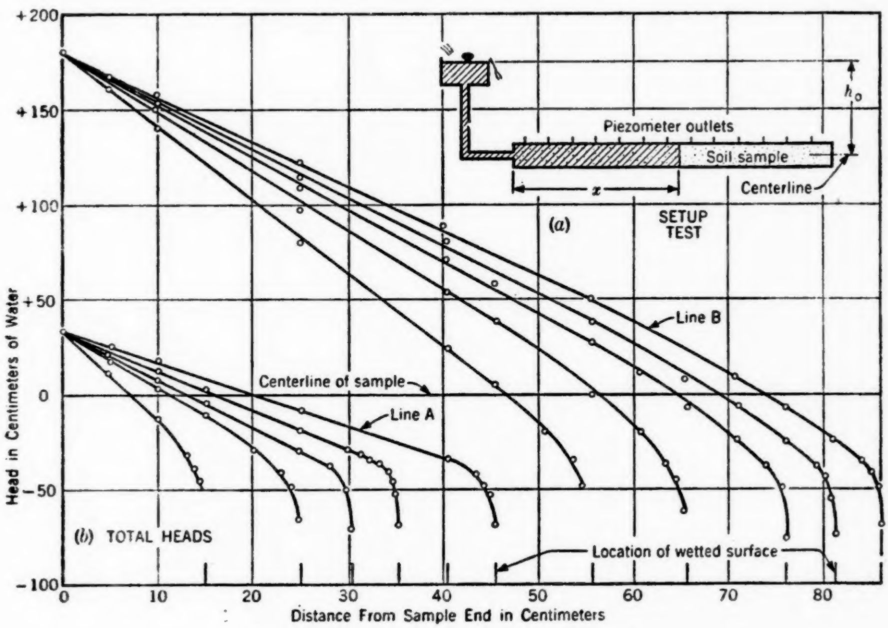

FIG. 3.—HORIZONTAL CAPILLARITY TEST IIIE

With the foregoing three differences in mind, the extension of description of flow processes from tubes to soil can better be understood.

Horizontal Capillarity Test.—Fig. 3(*a*) shows the setup for test IIIE which was a special laboratory soil test in which pore water pressures were measured. This test corresponded somewhat to the flow system for the tubes shown in Fig. 2. Water was admitted at the left end of the horizontal sample of soil under a static pressure of magnitude h_o. From the beginning of the given test until x = 49.8 cm, h_o was 35.0 cm; at x = 49.8, h_o was increased to

181.7 cm. Water pressures were measured at the piezometer outlets spaced every 5 cm along the length of the soil tube. Water pressures and tensions were read to an accuracy of 0.3 cm of water.[4] Fig. 3(*b*) is a plot of these water pressure readings taken at different values of x. For example, when the wetted surface has traveled to a distance of 45.5 cm, line A is the line connecting the total water heads shown on the vertical scale opposite the points at which they exist. Thus the piezometer 25 cm from the end of the soil read -8 cm and is so plotted on line A. The slope of line A is the gradient and corresponds to the plots of total head versus distance shown in Figs. 1 and 2.

To determine the varying degree of saturation along the length of a sample two similar tests were run in a segmented "lucite" tube. At any instant, this segmented tube could be sliced up to obtain the degree of saturation of the soil in each segment. The results of slicing up two samples are shown in Fig. 4

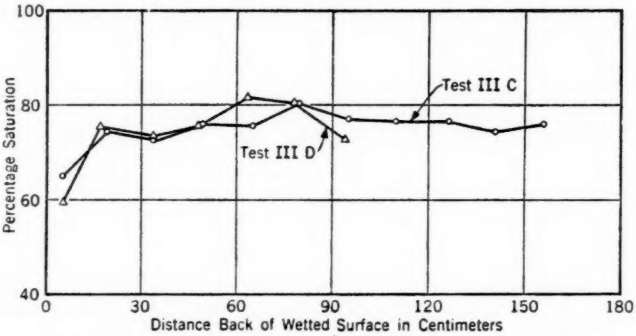

FIG. 4.—DEGREE OF SATURATION BACK OF WETTED SURFACE

which is a plot of the degree of saturation versus the distance back of the wetted surface. The only definite characteristic that can be noted in Fig. 4 is the lower degree of saturation just back of the wetted surface. This phenomenon could be observed clearly during the tests.

Fig. 4 illustrates difference (*a*) between the capillary tube analogy and the true soil action. Behind the large meniscus in the system shown in Fig. 2 the degree of saturation is 100%, whereas the corresponding degree of saturation in the sand tested is less than 80%. (Subsequent research[5] has shown that the numerical values in Fig. 4 are very close to those obtained for several other soils tested.) It has been thought by many engineers that the degree of saturation obtained by capillarity was very close to 100%, and in previous work in soil mechanics 100% saturation frequently has been assumed. The observed low value of "capillary saturation" points to inaccuracies in several currently used formulas and laboratory techniques of soil mechanics.

Theoretical Methods of Analysis of Horizontal Capillary Flow.—The currently used theory considers flow into soils to be similar to flow in one capillary

[4] "The Measurement of Pore Water Pressures in Cohesionless Soils," by T. William Lambe, *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, 1948, paper No. 3.18.

[5] "Experimental Investigation of the Degree of Saturation in Sands," by H. R. Parfitt and N. E. Pehrson, thesis presented to the Massachusetts Inst. of Technology, at Cambridge, in 1948, in partial fulfilment of the requirements for the degree of Master of Science.

tube as shown in Fig. 5. Also shown in Fig. 5 are the assumed distributions of head, degree of saturation, and permeability.

For water flowing in soil as shown in Fig. 5(a), the rate of flow,[6] q, by Darcy's law, is equal to

$$q = k\,i\,A = k\,\frac{h_o + h_c}{x}\,A \qquad (7a)$$

in which k is the Darcy coefficient of permeability; i is a gradient; A is the cross-sectional area; and h_o, h_c, and x are as shown in Fig. 5. The flow q can also be expressed as

$$q = \frac{dx}{dt}\,n\,A \qquad (7b)$$

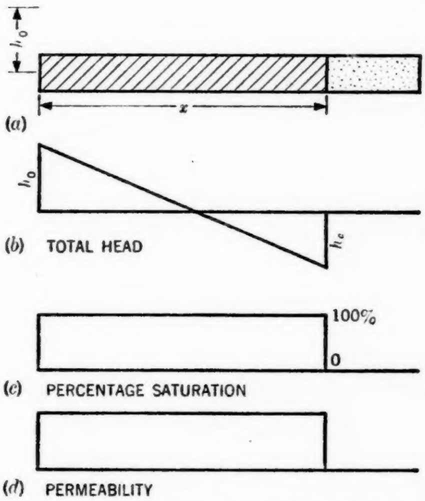

FIG. 5.—HORIZONTAL CAPILLARY FLOW AS ASSUMED IN PRESENT THEORY

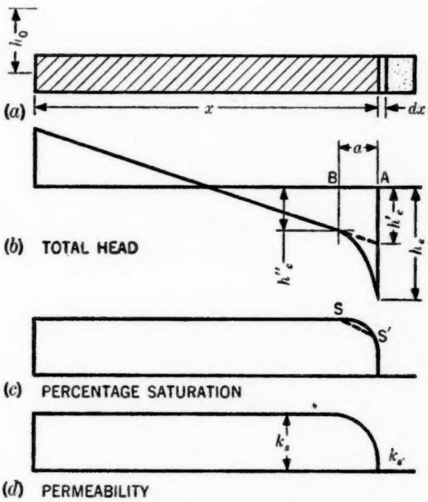

FIG. 6.—HORIZONTAL CAPILLARY FLOW AS ASSUMED IN PROPOSED THEORY

in which n is the porosity of the soil. Equating Eqs. 7,

$$k\,i\,A = \frac{dx}{dt}\,n\,A \qquad (8a)$$

or

$$k\,\frac{h_o + h_c}{x} = \frac{dx}{dt}\,n \qquad (8b)$$

The solution of Eq. 8b gives

$$\frac{\Delta(x^2)}{\Delta t} = \frac{2\,k}{n}\,(h_o + h_c) \qquad (9)$$

[6] "Capillary Phenomenon in Cohesionless Soils," by T. William Lambe, thesis presented to the Massachusetts Inst. of Technology, at Cambridge, in 1948, in partial fulfilment of the requirements for the degree of Doctor of Science.

Eq. 9 has been used as a basic equation for a laboratory test, called the horizontal capillarity test, to determine k and h_c. The slope of any (x^2 versus t)-curve has been taken as equal to $\frac{2\,k}{n}\,(h_o + h_c)$. By running the test at two different values of h_o, two equations of the form of Eq. 9 can be obtained; the simultaneous solution of the two equations gives values of the unknown soil properties k and h_c. In past years many tests run on several different soils in the Soil Mechanics Laboratory at the Massachusetts Institute of Technology (M.I.T.), in Cambridge, have shown that the slope of the (x^2 versus t)-curve is very nearly constant after the early stages of the test; but these tests have given values for k that could not be checked by other types of permeability tests. Also the head h_c obtained from such horizontal capillarity tests was lower than was thought reasonable.

A study of Figs. 3(b) and 4 offers an explanation of the aforementioned difficulties by revealing that the assumptions made in the foregoing derivation are questionable. The assumption of a degree of saturation equal to 100% is not justified and the assumption of a constant gradient is not valid in the region near the advancing wetted surface.

Based upon the ideas presented in the paper to this point, it is now possible to derive a more reasonable theory. Fig. 6 shows the same tube of soil as Fig. 5 with the plots of total head, h, degree of saturation, S, and permeability, k, as functions of the distance traveled. A comparison of Fig. 6 with Figs. 3(b) and 4 shows Fig. 6 to be in reasonable agreement with the actual distribution curves. By assuming in Fig. 6 that h'_c and a are constant, one can write, directly:

$$\frac{dx}{dt} = \frac{k_s}{S\,n}\,\frac{h_o + h'_c}{x} \qquad \text{(10)}$$

Integration of Eq. 10 gives

$$\frac{\Delta(x^2)}{\Delta t} = \frac{2\,k_s}{S\,n}\,(h_o + h'_c) \qquad \text{(11)}$$

Fig. 6(b) shows that h'_c and a actually depend on the gradient; therefore, Eq. 11, used for widely different gradients, would not give a good value for h'_c. A better assumption of distribution is that h''_c and a are constants; but the little additional refinement obtained by doing so does not justify the additional complexity. Eq. 11 appears almost identical with Eq. 9, but there are several important differences—namely:

1. Eq. 11 includes a term for the degree of saturation;
2. The permeability (Eq. 11) is that which occurs at a given value, S, the degree of saturation, and not the saturated value; and
3. The term h'_c is not a true capillary head, but is actually the value of water tension at the wetted surface required to give a straight-line gradient for the entire length of x.

To illustrate the use of Eq. 11, an application of it will be made to test IIIE. Fig. 7 is a plot of the squares of various x-values against the total elapsed

times, t, at which they were observed. The slopes of the (x^2 versus t)-curve are measured in square centimeters per minute and are, for h_o = 35.0 cm—

$$\frac{\Delta(x^2)}{\Delta t} = 11.2$$

and, for h_o = 181.7 cm—

$$\frac{\Delta(x^2)}{\Delta t} = 39.3$$

The flow in test IIIE was stopped at a value of x = 93.4 cm and the quantity of water, V_w, that had flowed into the soil at that point was 392 cu cm. These data and the porosity of 0.375 can be used to compute the average degree of saturation as: $S = \frac{V_w}{V_v} = \frac{V_w}{L\,n\,A} = 0.745$, in which V_w is the volume of water; V_v is the volume of voids; and L is the length of wetted soil. (The degree of

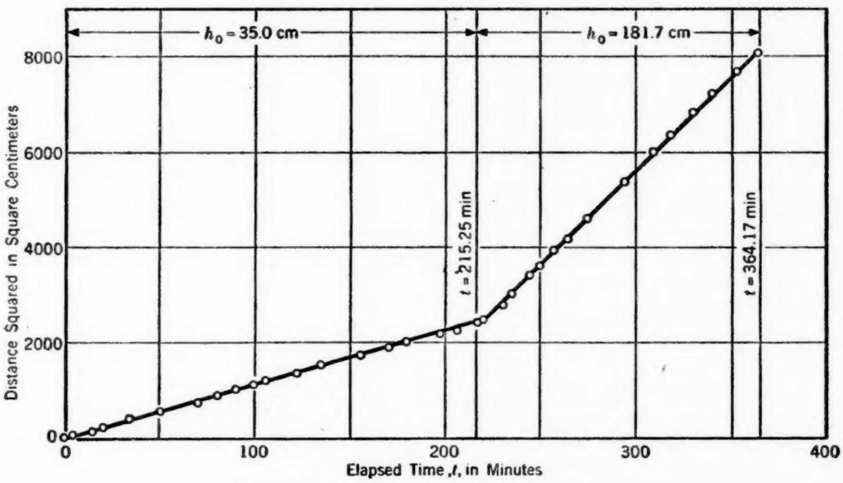

FIG. 7.—APPLICATION OF EQ. 11 TO TEST IIIE

saturation can also be determined by weighing the permeameters of soil in both the dry and wet states. The difference of these weights is the weight of pore water. Knowing the void ratio of the soil, one can easily compute the degree of saturation.) Since the degree of saturation is essentially constant for the distance from the left end of the soil to $x - a$, the value of S = 0.745 will be taken as constant. Substituting the foregoing data in Eq. 11 to obtain two simultaneous equations, and solving these equations, the result is a value of permeability which, reduced from the test temperature to 20° C, is $k\left(\substack{s=0.75\\T=20°C}\right)$ = 0.0254 cm per min, in which T is the temperature.

The results of permeability tests run on the tube of soil from test IIIE give, for $k\left(\substack{s=0.745\\T=20°C}\right)$:

(*a*) Constant head test; k = 0.0266 cm per min
(*b*) Falling head test; k = 0.0235 and k = 0.0264 cm per min

The differences in the foregoing values, obtained from the standard permeability tests, are due to different initial pressures in the entrapped air and represent the extremes that were obtained with the apparatus used. To compare, properly, the results of a horizontal capillarity test and a falling head test, one must consider in detail the pressure in the water and entrapped air initially and

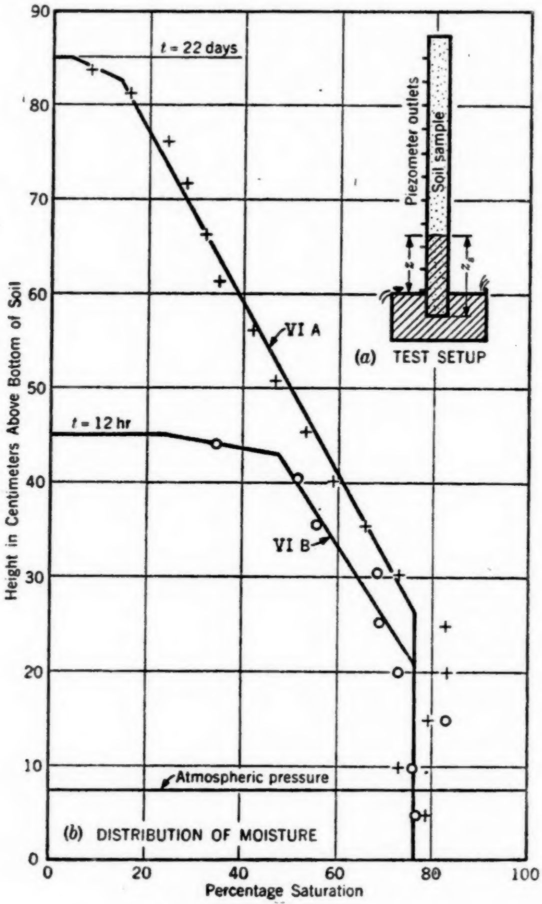

FIG. 8.—CAPILLARY RISE TEST

during each test. Nevertheless, one can see that the permeability from the horizontal capillarity test is in good agreement with the values determined from the other tests.

In summary it may be stated that the use of Eq. 11 with the data from a horizontal capillary test gives a good measure of the permeability of a soil that has been wet under the influence of capillarity. The value of capillary head obtained is an effective head and bears little relation to that determined by other laboratory capillary tests.

Capillary Rise Test.—The ideas developed for horizontal capillary flow are applicable in general to vertical flow, except that in vertical flow elevation head is more important. In capillary rise, the effect of elevation head is to decrease total head, thus reducing the rate of flow, and to make the range of low degree of saturation much larger and more important than in horizontal flow. No capillary tube analogy will be given for vertical flow because the important concept that flow in the small tubes precedes that in the larger ones, with the resulting larger gradients in the region just back of the wetted surface, can be carried forward from the previous analogy.

In order to obtain curves of the degree of saturation and the total water head versus the distance, a series of tests were run using the setup shown in

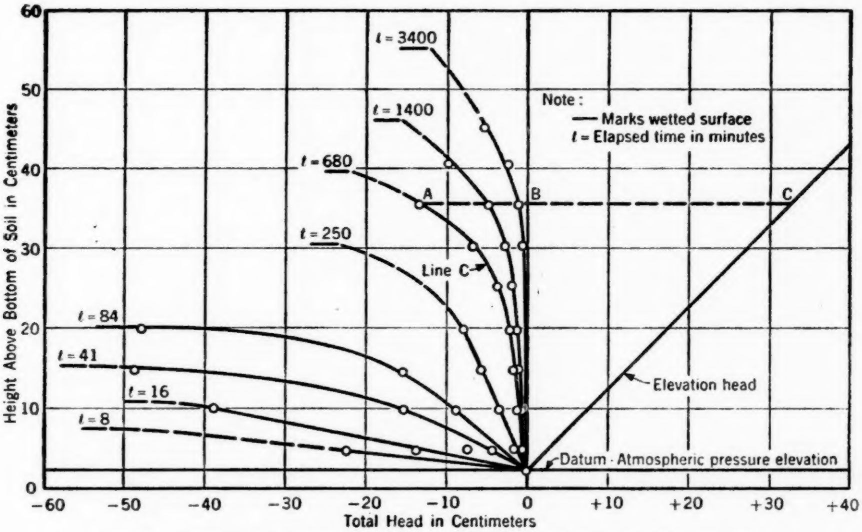

FIG. 9.—DEGREE OF SATURATION VERSUS DISTANCE ABOVE BOTTOM OF SOIL

Fig. 8(*a*). In these tests water from the constant head reservoir is permitted to rise by capillarity into the soil sample. During the test, the distance above the bottom of the soil, z_s, the water pressures at the piezometric outlets, and the elapsed time are recorded. Fig. 8(*b*) is a plot of the degree of saturation versus the distance above the bottom of the soil for two stages in the rise—that is, test VIB is after the wetted surface has risen 45 cm in an elapsed time of 12 hours and test VIA is after the wetted surface has risen 85 cm in an elapsed time of 22 days. The shape of these distribution curves seems to be characteristic. The abrupt end to the curve at the top is caused by stopping the capillary rise before a static condition has been reached. Long-time tests by another experimenter[7] indicate that the wetted surface would continue to rise at an infinitely small velocity for years and eventually rise to more than 100 cm for the type of soil used.

[7] "Capillarity Tests by Capillarimeter and by Soil Filled Tubes," by K. S. Lane and D. E. Washburn, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., Vol. 26, 1946, p. 460.

Fig. 9 is a plot of the measured pore water pressure versus the height above the bottom of the soil for various positions of wetted surface in its process of rising. (The broken line part of each curve was obtained by extrapolating back to intersect the wetted surface elevation line.) For example, at an elapsed time of 680 min when the wetted surface has risen 39.5 cm from the bottom of the soil, line C is the locus of total head in the soil sample. Thus, at a point (point A, Fig. 9) 35.5 cm above the bottom of the soil, the total head (AB) is -13.2; the elevation head (BC) is $+33.0$; and the pressure head (AC) is -46.2 cm of water. The slope of line C, or the head lost in a given height, is the gradient in the sample at time, $t = 680$ min; line C in Fig. 9 corresponds to line A in Fig. 3(*b*).

Fig. 9 shows a very important point which has not received proper appreciation in the past. A large part of the available head difference between the wetted surface and the atmospheric pressure line is lost in the region just behind the wetted surface. Remembering the previous discussion under horizontal flow, one should not be surprised to find that the gradient is far from constant. The importance of this point will be emphasized in the following presentation of theoretical methods.

Theoretical Methods of Analysis of Capillary Rise.—Since the conventional method of computing the rate of capillary rise is presented elsewhere,[8] only

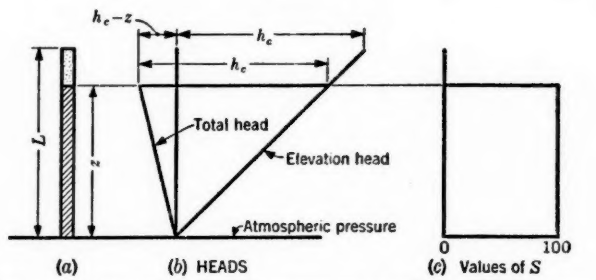

Fig. 10.—Capillary Rise As Assumed in Currently Used Theory

its derivation will be outlined. The basic assumptions used in the conventional method are implied in Fig. 10 in which water is shown rising by capillarity in a tube of soil. The position of the wetted surface for this capillary rise is located by its distance z above the atmospheric pressure line. Making use of the assumed distributional curves shown in Fig. 10 in Darcy's law,

$$q = k\,i\,A = k\,\frac{h_c - z}{z}\,A \quad \text{(12a)}$$

in which h_c is defined as the maximum height to which capillary water will rise. Also,

$$q = v\,A = \frac{dx}{dt}\,n\,A \quad \text{(12b)}$$

in which v is the velocity of movement of the wetted surface. Equating Eqs.

[8] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1944.

12 and integrating,

$$t = \frac{n\,h_c}{k}\left(1\,n\,\frac{h_c}{h_c - z} - \frac{z}{h_c}\right) \quad \text{(13)}$$

Eq. 13 is used for computing the rate of capillary rise, in which the following assumptions have been made or implied:

(1) The degree of saturation of all wetted soil is 100%;
(2) The permeability is the saturated permeability; and
(3) The gradient is uniform.

The test data in Fig. 8(*b*) prove assumption (1) to be incorrect; and, since the permeability is a function of the degree of saturation, assumption (2) is also incorrect. In view of the previous observation, Eq. 12*a* is more accurately interpreted as applicable to only the bottom of the sample. Since, for small heights, the degree of saturation seems to be essentially constant after the first few minutes, the permeability remains almost constant at the bottom. The permeability to be used in Eq. 13 is the value for a degree of saturation of about 80% (see Fig. 8(*b*)). (For the soil used in these tests, the permeability at 80% saturation is about a third of the value when the saturation is 100%.) Assumption (3) is even less valid than assumptions (1) or (2), as Fig. 9 clearly shows.

The writer has developed[6] a theory which is founded on more reasonable assumptions than is Eq. 13. This theory, based on the use of an effective capillary head, is not presented here for several reasons: First, Fig. 9 shows that about half of the capillary rise occurred while the gradient at the bottom of the soil was less than a few hundredths. This small gradient not only is approximately equal to the experimental accuracy of the laboratory apparatus used to measure it, but also is so small that the results of an actual capillary test would be needed to indicate how it varies; and, second, if a more correct expression is used for the gradient, the resulting formula for the rate of rise becomes so complicated as to be of little value.

The use of Eq. 13 to predict the elapsed times for the various stages of capillary rise may yield values that are many times too small. This is as expected since the permeability and the gradient used in Eq. 13 are both too large.

Drainage of Cohesionless Soil

Description of Drainage.—As in the case of capillary flow into soils, the description of the drainage process of cohesionless soils can be aided by first studying the drainage of simple systems of capillary tubes. In Fig. 11 are shown two capillary tubes of diameter D; the tubes are connected to each other at many points. Initially, the tubes are completely filled for their entire length, L, flow being prevented by closed valves at elevation A.

Drainage of the two tubes is started by opening the valves at point A and maintaining a free water surface there. Upon opening the valves the pressure head at all points disappears, thereby giving a total head distribution equal to the elevation head distribution. Water begins to drain because of the difference of total head between adjacent points. When the total head becomes equal to

the elevation head the gradient is L/L or 1. After a small amount of flow occurs, menisci are developed at the receding water-air interface. The menisci apply a capillary head or tension to the water, giving a total head distribution shown by line DB in Fig. 11. The elapsed time required to obtain the head distribution BD is negligible and is taken as zero.

The plot of heads versus depth in Fig. 11 is a complete picture of all heads at any stage of drainage. Thus, at time, t, the water level has fallen to elevation, z. A horizontal line was drawn from elevation z in the tube to intersect line DE and give point F. Line DE was obtained by drawing a line parallel to BD but offset a horizontal distance of h_c to the left of the elevation head line. The elevation head at time t is $\overline{HG}$; the pressure head is $-\overline{FG}$; and the total head is $\overline{HF}$. A line from points F to B represents the distribution of total head in the water in the tubes at time t, and the reciprocal slope of FB is the gradient. Since every other factor in the Poiseuille law is constant in this

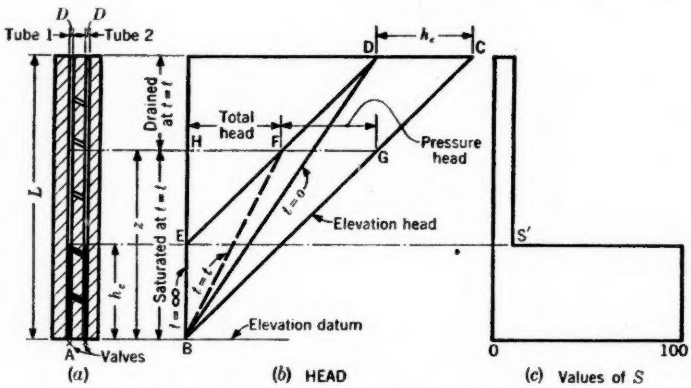

Fig. 11.—Drainage of Capillary Tubes, System 1

case, the rate of fall of the water line is proportional to only the gradient i. After the menisci are developed, the gradient varies from a maximum of $L - h_c/L$ as indicated by the inverse slope of line BD to $z - h_c/z$ at time t, and finally to zero as indicated by the inverse slope of line BE. Therefore, the water line falls at an increasingly slower rate to its final position, h_c, above the datum.

The final distribution of water in the two-tube system is shown in Fig. 11 by a plot of the degree of saturation, S, versus the height. To a height, h_c, above elevation A, the tubes are completely filled with water, giving a degree of saturation of 100%. From an elevation of h_c to an elevation of L, the only water present is the water entrapped in the lateral connecting system, giving a degree of saturation equal to S'. Since this entrapped water is not continuous, S' is not a function of the elevation. In this system there is no intereffect of the flow in the two tubes because the tubes are the same size.

Although the system in Fig. 11 is like an actual draining mass of soil, it is different in several important respects. One of these differences is that in soils the tubes are not of the same size, but vary over a large range. System 2,

Fig. 12, goes one step further than that of Fig. 11, in having tubes of diameters D_S and D_L; in every other respect the tube setup in Fig. 12 is like that in Fig. 11.

During the drainage of system 1, the water levels in the tubes were always at the same elevation. In system 2, after the beginning of flow, the two levels will never be together since the tubes are of different size. Until menisci are developed, the draining processes in the two systems are similar. Because it is larger in diameter, tube 2 will develop its capillary head fully before tube 1 does. Thus, at time, $t = 0+$, the total head line $\overline{BD}$ is determined by tube 2. The water level in tube 2 will fall to elevation z_1 before the full capillary head at the top of tube 1 is mobilized. Until h_{c1} is mobilized, there is no movement of the water level in tube 1. The condition of incipient movement of the level

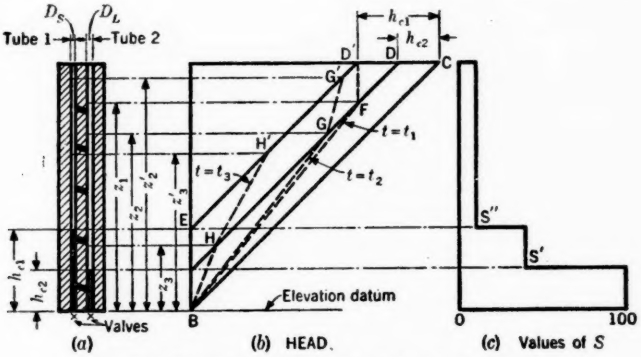

Fig. 12.—Drainage of Capillary Tubes, System 2

in tube 1 is represented in Fig. 12 as that occurring at $t = t_1$. Line BF represents the total head distribution to the elevation z_1; from z_1 to L the total head is constant, as shown by FD′.

At time $t = t_2$ the level in tube 2 is at z_2, and in tube 1, at z'_2. Line BG represents the total head distribution to elevation z_2 and GG′ from z_2 to z'_2. The gradient from z_2 to z'_2 is less than that below z_2.

The rate of movement, q, of water in either tube is proportional to $D^2 i$ (Poiseuille's law) since all other factors are constant in this setup. Thus, the rate of fall in tube 2 continues to be faster than that in tube 1 until i_2 is equal to the gradient above the line of saturation, multiplied by $\frac{D^2_S}{D^2_L}$. Position 3, as indicated by z_3 and z'_s, represents a further stage of drainage in which the gradient above the lower level is greater than below. It can thus be seen that gradients above and below the lowest water level would be equal to each other only by coincidence.

Before the analysis is carried from the tubes into the soils, a number of conclusions, based on the preceding discussion of the three analogies, will be stated:

1. Drainage proceeds at an increasingly slower rate due to the gradient, which decreases with time;

2. In a system of two tubes with different diameters, as in system 2, the larger tube has the greater influence on the rate and the quantity of drainage; and

3. The water held in the tubes above the elevation of the maximum capillary head is not continuous, but is water entrapped in the cross connections and, therefore, the quantity retained is not a function of the elevation.

In addition to the differences between tubes and soils, listed in the paragraph following Eq. 11, there is another difference that can be important in drainage —natural soils can be partly saturated from the beginning. Entrapped air causes time lags for changes in water pressure to occur; if the entrapped air is not distributed uniformly the gradient is affected.

Drainage Test.—The results of a drainage test on an initially saturated sample are plotted in Fig. 13. The setup for this test was similar to that for

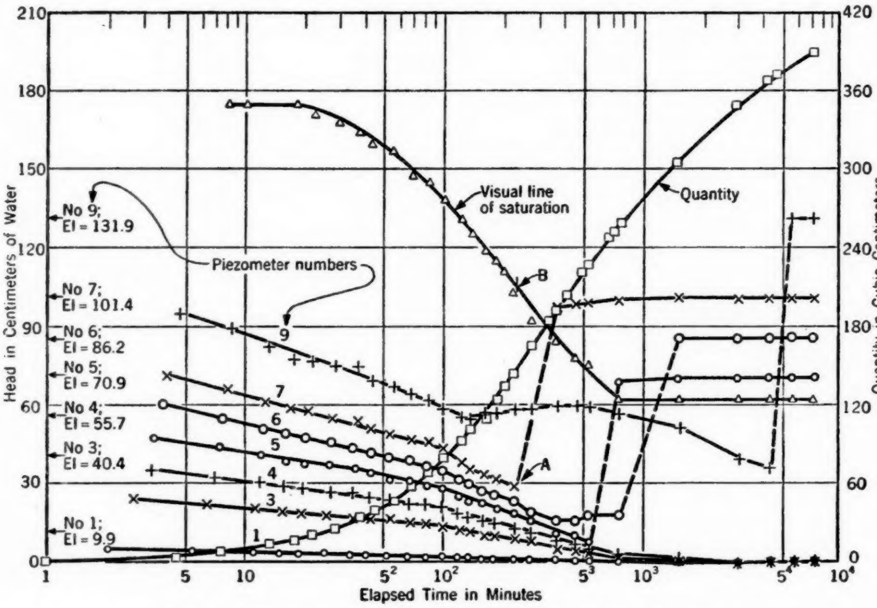

FIG. 13.—DRAINAGE TEST IF ON INITIALLY SATURATED SAMPLES

the capillary rise shown in Fig. 8(*a*); the elevation of the piezometer outlets is shown on the left-hand scale in Fig. 13. For example, at an elapsed time of 225 min, the total head at piezometer No. 7 was 29 cm (shown by point A, Fig. 13). The pressure head at piezometer No. 7 is the total head minus the elevation head or $29 - 101 = -72$ cm. Since point A is located where the water became discontinuous at piezometer No. 7, the value of -72 cm is the maximum water tension measured, or the maximum capillary head measured, at piezometer No. 7. At an elapsed time of 225 min, the curve marked "visual line of saturation" shows the line of saturation to be approximately at the

elevation of piezometer No. 7 (point B, Fig. 13). The "quantity" curve is a plot of the cumulative quantity of flow at any time.

In Fig. 14 the pressure data from Fig. 13 are plotted in a manner similar to the water pressure in the tubes shown in Figs. 10 and 11. The inverse slope of the lines connecting the plotted pressure data (Fig. 13) is the gradient. Point A, Fig. 13, can be seen in Fig. 14 with its total head of aA, elevation head of ab, and pressure head of −bA.

After test IF was run until practically all drainable water seeped out, the sample was sliced up and the degree of saturation of the soil segments was

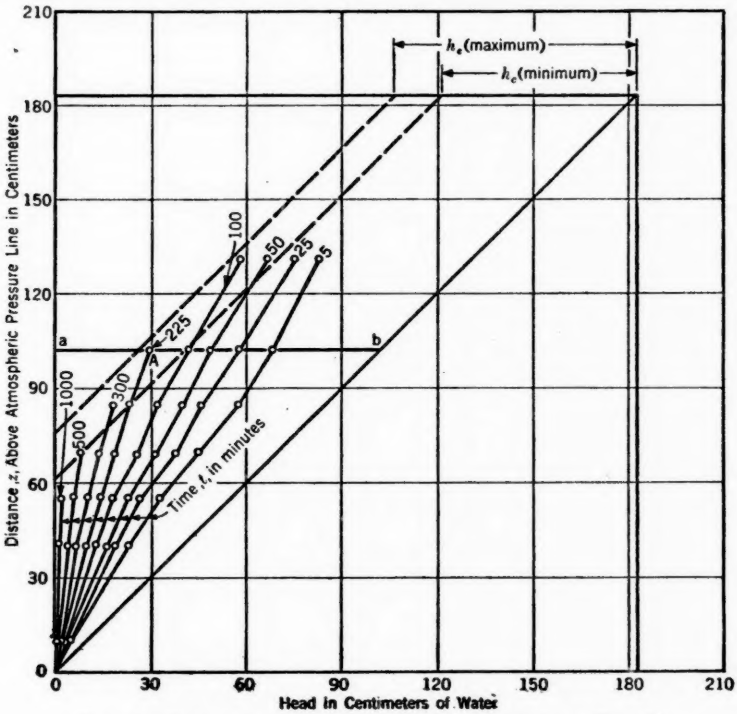

FIG. 14.—DISTANCES ABOVE ATMOSPHERIC PRESSURE LINE, TEST IF

obtained. Fig. 15 is a plot of the degree of saturation versus the distance above the atmospheric pressure.

A study of Figs. 13, 14, and 15 (which present actual test data) shows that the true drainage process can be well explained by the simple, capillary tube analogies shown in Figs. 11 and 12. The differences between the action of the analogy and the actual process are attributed to the previously listed differences in their physical make-ups.

Two interesting conclusions can now be drawn:

(1) A considerable amount of drainage occurs after the visual line of saturation appears to have reached its ultimate position (Fig. 13); and

(2) The visual line of saturation is not the true line of saturation (Fig. 15).

The drainage of a partly saturated soil is of much practical interest, since as shown in connection with Fig. 4, a soil "saturated" by capillary action may have a degree of saturation considerably less than 100%. A test (test IG) was run on the soil sample from test IIIE which was a horizontal capillary flow test, and thus the sample had an average degree of saturation of 79%. This test reveals two main differences between the drainage of a saturated and partly saturated soil mass. The first difference is the lower total ultimate quantity drained from a partly saturated soil. This difference follows from the fact that, since all the voids of the soil are not filled with water, less water will flow out. The second difference is the slower rate of flow and the slower rate of head dissipation in the partly saturated soil. The rate of flow of water is lower because the permeability is decreased by entrapped air.

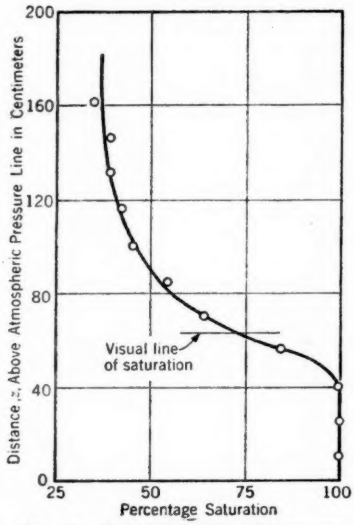

Fig. 15.—Distribution of Moisture in Drainage

There is another time lag in the drainage of partly saturated soils which is due to the time required for adjustment to changes of water pressure. Unlike water, air is very compressible; therefore, in order that air may adjust itself to a difference in pressure it must compress or expand. This volume change in the air necessitates a flow of water which in turn requires time. Thus, when the menisci are first developed, the tension is not applied suddenly throughout the water, for the reason described. Although, in drainage, this time lag may be of minor importance, it becomes of major importance in laboratory tests on partly saturated soils.[4]

A comparison of the drainage curves of two samples of soil is afforded by Fig. 16. One sample was completely saturated and the other partly saturated at the beginning of the test. In this comparison, the volume of water drained divided by the volume of voids is plotted against the elapsed time multiplied by the permeability of the soil. The agreement of the two curves in the early stage signifies that, in this early stage, the rate of drainage is proportional to the permeability.

Theoretical Methods of Computing Drainage.—A number of methods of computing drainage has been analyzed and evaluated; and solutions from the various theoretical methods were compared with the drainage curve of test IF. The theoretical evaluation and comparison with test results[9] showed all theoretical methods studied to be approximate. A major problem in the use of the theoretical methods is the proper selection of soil properties.

Capillary Heads

General.—Much of the literature implies that a given soil under given conditions possesses a single, definite capillary head, as does a capillary tube. Re-

[9] Discussion by T. William Lambe of "Investigation of Drainage Rates Affecting Stability of Earth Dams," by F. H. Kellogg, *Transactions*, ASCE, Vol. 113, 1948, p. 1294.

cently, to this concept was added the one which considered a soil to possess two capillary characteristics called the "active capillary rise" and the "passive capillary rise." To accompany this concept, a number of laboratory tests have been devised to measure the "active rise" and "passive rise," with little agreement being reached between the values obtained in the several types of tests. Much of the confusion that now exists is due to the oversimplification introduced by comparing the movement of capillary water in soils to that of water in a bundle of nonconnected capillary tubes. This paper shows that the capillary characteristics of a given soil cannot be completely represented by one or two head values; several capillary heads are required for adequate representation.

The Range of Capillary Heads.—Fig .17 is a plot of the degree of saturation versus the height above the atmospheric pressure line for test IF, a drainage

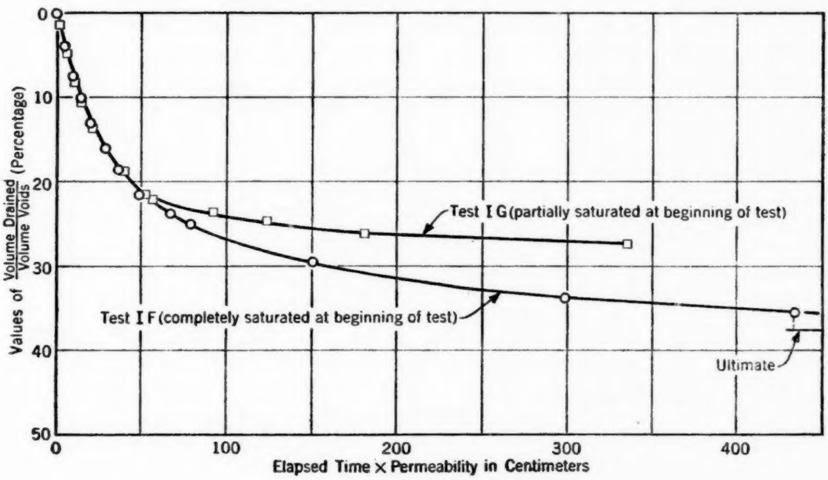

FIG. 16.—DRAINAGE OF PARTLY AND COMPLETELY SATURATED SOILS

test on a sample that was initially saturated, and test VIA, a capillary rise test on a sample that was initially dry. It would seem logical that point A (Fig. 17) (which is approximately the point where the curve ceases to be vertical) is the highest elevation at which there exists any continuous channel of water from the free water surface below. Therefore, the distance from point A to the free water surface is taken as the maximum capillary head, h_{cx}. Another critical point on the degree-of-saturation curve for a drainage test is the highest elevation at which complete saturation exists (point B, Fig. 17). The distance from the free water surface to this point is called the saturation capillary head, h_{cs}.

On the distributional curve from the capillary rise test, there are two critical points. The distance from the free water surface to the highest elevation to which capillary water rose (point C, Fig. 17) is called the capillary rise, h_{cr}. The distance from the free water surface to the highest elevation at

which the maximum capillary degree of saturation exists (point D, Fig. 17) is named the minimum capillary head, h_{cn}.

The four capillary heads are limits of the possible range of capillary heads that a soil can have. Any capillary head associated with drainage must lie between h_{cx} and h_{cs}—similarly, any associated with capillary rise must lie between h_{cr} and h_{cn}. Since it is the size of void at the air-water interface which

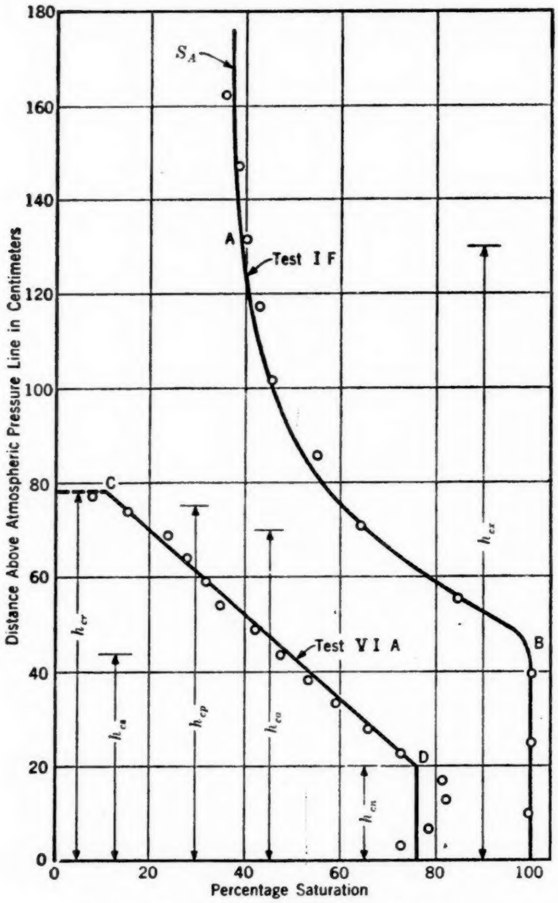

FIG. 17.—RANGE OF CAPILLARY HEADS

determines the capillary head, it is possible for a void to support water that filled larger voids below its surface, yet not raise the water past these larger voids. Therefore, h_{cx} is greater than h_{cr}, and h_{cs} is greater than h_{cn}, as might be expected.

In addition to the previously described limiting capillary heads, two others are shown in Fig. 17. One of these is a weighted, average capillary head, h_{ca}, selected to make $(L - h_{ca})\ n\ A\ (100 - S_A)$ equal to the quantity of water

drained. The other capillary head was obtained by cutting off the water supply in a horizontal capillarity test (test IIIE) and measuring the water tension that developed. By cutting off the water supply, the movement of water was retarded but not prevented since the smallest voids stole water from the larger voids. It is to be noted that the maximum water tension measured (75 cm, or h_{cp}) is a conservative measure of the capillary rise of 78 cm.

Since flow was greatly slowed down an approximate value of the maximum capillary pull could be obtained; but h_{cp} would not be equal to h_{cr} unless the flow were absolutley prevented.

The Effective Capillary Head.—Between the two extremes, h_{cx} and h_{cn}, there exists an infinite number of capillary heads. In any soil problem involving capillarity the value of capillary head that is effective is a value within the range of the limiting values. The effective value of capillary head to be used in any problem depends on the particular problem. The many problems involving capillarity in soils can be divided into three classes as follows:

(*a*) The determination of the height above free water surface to which capillary water will rise or be retained;
(*b*) The determination of the rate of movement of capillary water; and
(*c*) The determination of the quantity of water that rises or is retained at any height.

The selection of an effective capillary head for problems of class (*a*) is the simplest. The height at which continuous water will be held by capillarity is h_{cx}; the height to which capillary water will rise is h_{cr}. (The value of h_{cr} shown in Fig. 17 would be larger, probably about 100 cm or 110 cm had test VIA been of several years' duration instead of 22 days.[9]) Recent inspection[10] of many airfields has led to the conclusion that it is nearly always h_{cx} rather than h_{cr}, that is the maximum height to which continuous water exists. The explanation of this fact is that the rain water, or other runoff, percolates down from above.

The selection of an effective capillary head for problems involving rate of flow is much more difficult than the selection for problems of class (*a*). In draining a soil, the tension back of the retreating menisci in the large voids is approximately h_{cs}, whereas that in the small voids approaches h_{cx}. The selection of an effective capilllary head in a drainage formula depends upon the nature of the particular formula.

In computing the rate at which water flows into a soil by capillarity, the selection of an effective capillary head again depends on the conditions inherent in the formula used. Since all the methods discussed in this paper assume a uniform gradient, the effective capillary head to be used is a value that gives a straight-line gradient which causes the same rate of flow as the true variable gradient. One type of effective capillary head can be obtained from the results of a horizontal capillarity test in which two magnitudes of applied head are used; from test IIIE the effective capillary head was found to be about 24 cm. This capillary head is fictitious and thus cannot be shown with meaning in Fig. 17.

[10] "Report on Frost Investigation," New England Div., Corps of Engrs., War Dept., Boston, Mass., April, 1947.

The effective capillary head used for computing the corresponding rise is also a fictitious value which gives an equivalent uniform gradient. As there is no applied pressure in the capillary rise process to keep the flow in the large voids near that in the small voids, the gradient is variable and there is no range of height in which it approaches a constant except that below point D (Fig. 17). Because of this fact the development of a formula for the rate of capillary rise is a most difficult problem. As seen in Fig. 9, for the early part of the rise, the effective capillary head was about 40 cm. Above $z = 40$ cm, the effective capillary head, based on an assumed uniform gradient, becomes larger. Again the effective capillary head of 40 cm cannot be shown, with meaning, in Fig. 17.

The selection of an effective capillary head for problems of class (*c*), or the determination of the quantity of water that will rise or be retained at any given height, can be easily made from curves such as are shown in Fig. 17. Since problems of class (*c*) are distributional problems, they can be solved better when curves of the degree of saturation versus the elevation, rather that any particular values of capillary head, are available. For example, if, in any given problem, it is known that a degree of saturation of 60% or more is not desirable at the surface of the soil, a depth of 75 cm or more of soil must exist above the free water surface. Likewise, if the requirements of any other problem are known, the effective capillary head can be easily taken from a plot of the degree of saturation versus the elevation which was the procedure by which the 75-cm value was obtained in the foregoing example.

The long time required to develop h_{cr} and h_{cs} fully must be kept in mind. It was noted that the capillary head determined by the large voids in drainage, or the capillary head effective in the initial gradient, was greater than h_{cs}. This is explained by the considerable time allowed for the tortuous flow necessary to develop h_{cs}. Also all indications point to the probability that the maximum capillary head during drainage is less than h_{cx} because of the low velocity of flow required to develop h_{cx}. The previous statements as to when large and small voids are effective depend on the rate of flow.

Laboratory Tests to Determine Capillary Head.—Most laboratory tests determine only two values of capillary head, h_{cr} and h_{cs}, known as "active capillary rise" and "passive capillary rise," respectively. For a time, some engineers thought the two should be equal. A research project[11] reported in 1944 was aimed at finding the relation between the two "rises."

The current terminology of capillary heads seems inappropriate. The use of the terms "active" and "passive" is reasonable, but to add the word "rise" to each is not reasonable. Certainly there is nothing about h_{cs} which justifies it being called a rise, in fact, "passive" and "rise" imply opposite meanings. In view of the field observations[10] which indicate that soils obtain their capillary water by infiltration from above, one may question the value of obtaining the magnitudes of h_{cr} or h_{cn} except for comparative studies. Certainly the full magnitude of h_{cr} would have little meaning, since it is hard to conceive a soil deposit not being subject to water from above for as long a period as is required

[11] "Capillarity in Sands," by Raul Valle-Rodas, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., Vol. 24, 1944, p. 389.

to develop h_{cr}. In general, it can be concluded that h_{cr} is one of the least used and one of the most difficult to obtain of the capillary heads.

The h_{cs}-value is the one determined by Gunnar Beskow[12] and is one of the less difficult values to find. Also the practical use of h_{cs} to determine the height to which a soil mass will retain its maximum degree of saturation makes h_{cs} more important than h_{cr}.

The maximum capillary head has importance, but for a soil such as the one used in this project having a degree-of-saturation versus an elevation curve which does not break sharply at the end of the continuous water, the exact numerical value of h_{cx} is not too important. For example, it does not make much difference if h_{cx} is taken as 100 cm instead of 130 cm because there is very little continuous water in the range from 100 to 130.

The most meaningful and useful capillary head is h_{ca}. This value of capillary head reflects h_{cx} and h_{cs} plus the distribution between them and is thus a more indicative property of the soil than any of the other values of capillary head. If a single value of capillary head were to be adopted as a soil property and called "the capillary head of the soil," it probably should be h_{ca}. A test similar to test IF would give h_{cx}, h_{cs}, h_{ca}, and the distribution of retained water. Although a drainage test in a tube that could be sliced up is not as easy to run as a capillarity test in a device similar to that devised by Mr. Beskow, it is certainly not as time consuming as the capillary rise test.

Conclusion

It is suggested that the term "capillary head" be used in place of "capillary rise"; and that at least five meaningful capillary heads be recognized (Fig. 17) as: h_{cx}, h_{cn}, h_{ca}, h_{cr}, and h_{cs}. Also, the development of standard capillary tests should be considered with serious thought given to the adoption of some kind of drainage testing apparatus as standard laboratory equipment. The development of a test to measure h_{cp} as an indication of h_{cr} should be studied.

Again it is pointed out that all the research reported herein was performed on one soil under one set of test conditions. There is a great need for much more research on the subject of capillarity. Many of the trends suggested in this paper should be studied in detail on several soils under different conditions. There may be some definite relationship between various capillary heads; if such a relationship exists, it may be possible to predict several of the capillary heads from one.

Acknowledgment

The writer gratefully acknowledges the work of Frederick F. Flanders; his mechanical skill made possible the construction of special apparatus without which the research work reported in the paper would have been seriously retarded. M. J. Holley, Jr., Assoc. M. ASCE, and Victor F. B. De Mello, Jun. ASCE, of M.I.T. deserve thanks for reading a draft of this paper and making a number of helpful suggestions. The writer expresses his appreciation to D. W. Taylor, Assoc. M. ASCE, associate professor of soil mechanics at M.I.T., for his aid in conducting the reported research and in preparing this paper Mr. Taylor's suggestions and criticisms led to solutions that may not hav otherwise been reached.

[12] "Soil Freezing and Frost Heaving with Special Application to Roads and Railroads," by Gunnar Beskow, Technological Inst., Northwestern Univ., Evanston, Ill., 1935.